

CBCS SCHEME

USN

--	--	--	--	--	--	--	--	--	--

21MAT41

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025

Complex Analysis, Probability and Statistical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Define analytic function and derive Cauchy–Riemann equations in polar form. (06 Marks)
- b. If $f(z)$ is analytic, show that :
$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] |f(z)|^2 = 4|f'(z)|^2. \quad (07 \text{ Marks})$$
- c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z+2)} dz$, where c is the circle $|z| = 3$. (07 Marks)

OR

- 2 a. Show that $f(z) = z + e^z$ is analytic, and hence find its derivative in terms of z . (06 Marks)
- b. Find the analytic function $f(z) = u + iv$, given that $u = x^2 - y^2 + \frac{x}{x^2 + y^2}$ by Milne Thomson method. (07 Marks)
- c. State and prove Cauchy's integral formula. (07 Marks)

Module-2

- 3 a. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. (06 Marks)
- b. Obtain the series solution of Bessel's differential equation :
$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0. \quad (07 \text{ Marks})$$
- c. Express $x^3 - 5x^2 + x + 2$ in terms of Legendre polynomials. (07 Marks)

OR

- 4 a. Obtain the series solution of Legendre differential equation :

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$

(06 Marks)

- b. Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomial.

(07 Marks)

- c. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

(07 Marks)

Module-3

- 5 a. Find Karl Pearson's coefficient of correlation for the following data :

x	10	14	18	22	26	30
y	18	12	24	6	30	36

(06 Marks)

- b. Fit a straight line $y = ax + b$ for the data :

x	5	10	15	20	25
y	16	19	23	26	30

(07 Marks)

- c. Obtain the lines of regression and hence find the coefficient of correlation for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(07 Marks)

OR

- 6 a. Ten students got the following percentage of marks in two subjects x and y. Compute their rank correlation coefficient.

Marks in x	78	36	98	25	75	82	90	62	65	39
Marks in y	84	51	91	60	68	62	86	58	53	47

(06 Marks)

- b. Compute the means \bar{x} , \bar{y} and the correlation coefficient r from the given regression lines,

$$4x - 5y + 33 = 0$$

$$20x - 9y = 107.$$

(07 Marks)

- c. Fit a Parabola $y = ax^2 + bx + c$ by the method of least squares for the following data :

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

(07 Marks)

Module-4

- 7 a. A random variable X has the following probability function :

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

Find K.

Also find :

i) $P(x \geq 5)$

ii) $P(3 < x \leq 6)$.

(06 Marks)

- b. Derive the mean and variance of Poisson distribution.

(07 Marks)

- c. The probability that a pen manufactured by a factory be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the probability that :

i) Exactly two are defective

ii) Atleast two are defective

iii) None of them are defective.

(07 Marks)

OR

- 8 a. A random variable X has the density function :

$$f(x) = \begin{cases} kx^2 & -3 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Evaluate K.

Also find :

i) $P(1 \leq x \leq 2)$

ii) $P(x \leq 2)$

iii) $P(x > 1)$.

(06 Marks)

- b. The probability that an individual suffers a bad reaction from as injection is 0.001. Find the probability that out of 2000 individuals.

i) Exactly three

ii) More than 2 will get bad reaction.

(07 Marks)

- c. In a normal distribution 31% of the items are under 45 and 8% over 64. Find the mean and S.D. Given $A(0.5) = 0.19$ and $A(1.4) = 0.42$.

(07 Marks)

Module-5

- 9 a. The joint probability distributions of two random variables are given below :

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Determine :

- Marginal distributions of x and y
- $E[X]$ and $E[Y]$
- Verify X and Y are independent.

(06 Marks)

- b. Define :

- Null hypothesis
- Type – I and Type – II errors
- Level of significance.

(07 Marks)

- c. A certain stimulus administered to each of 12 patients resulted in the following changes in the blood pressure, 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that stimulus will increase the blood pressure [$t_{0.05}$ for 11 d.f is 2.201].

(07 Marks)

OR

- 10 a. The joint probability distribution of two random variables X and Y are given below :

X \ Y	-3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Determine :

- $E(X)$ and $E(Y)$
- $E[XY]$
- $COV(X, Y)$.

(06 Marks)

- b. Find the student 't' test for the following variable values in a sample of 8 are -4, -2, -2, 0, 2, 2, 3, 3. Taking the mean of the universe to be zero.
- c. The theory predicts the proportion of beans in the four group A, B, C and D should be 9 : 3 : 3 : 1. In an experiment among 1600 beans, the number in the four groups were 882, 313, 287 and 118. The goodness of fit χ^2 values of above data is approximately equal to? ($\chi^2_{0.05} = 5.99$).

(07 Marks)

(07 Marks)
